Introduction to Crystallography and Mineral Crystal Systems
by Mike and Darcy Howard

Part 3: The Cubic (Isometric) System

Now that you have read the two previous articles, you are ready to consider the first of our 6 crystal systems. So let's begin.

There are 15 forms, all closed, in the ISOMETRIC CRYSTAL SYSTEM—more than in any other single system we will examine. You may wish to briefly refer back to the first article in this series, when we built an axial cross. In the isometric system, all 3 crystallographic axes are at right angles to each other and are the same length. The axes are renamed a1, a2, and a3. **We need to remember that a3 is vertical, a2 is horizontal, and a1 is front to back.**

Crystal forms in the isometric system have the highest degree of SYMMETRY, when compared to all the other crystal systems. Did you know that there is only ONE object in the geometrical universe with perfect symmetry? Consider the sphere (fig. 3.1). Infinite planes of symmetry pass through its center, infinite rotational axes are present, and no matter how little or much you rotate it on any of its infinite number of axes, it appears the same! A sphere is the HOLY GRAIL of symmetry!! No crystal system even approaches a sphere's degree of symmetry, but the isometric system is often quickly recognizable because some of the forms and combinations of forms somewhat approach sphericity (or, at least, roundness), especially when the faces begin to be curved, due to the high degree of symmetry in the isometric system.

Let's begin by looking at the Hermann-Mauguin notation for the first seven isometric forms and each form's notation:

<table>
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<tr>
<th>Cube {001}</th>
<th>Dodecahedron {011}</th>
<th>Trapezohedron {hhl}</th>
<th>Hexoctahedron {hkl}</th>
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<tr>
<td>Octahedron {111}</td>
<td>Tetrahexahedron {0kl}</td>
<td>Trisocahedronv {hll}</td>
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For these forms, the 3 crystallographic axes are axes of 4-fold rotation. There are also 4 diagonal axes of 3-fold rotary inversion that pass through the form at the point where the cube's 3 faces would join. Furthermore, there are 6 directions of 2-fold symmetry (at the center of the line formed by the intersection of 2 planes). There is also a center of symmetry. There are 9 mirror planes (see figs. 1.5 and 1.6 in first article in this series). This combination of symmetry elements defines the highest possible symmetry of crystals. So the Hermann-Mauguin notation is 4/m-32/m.

In a textbook, my notation (-3) is presented as the number 3 with a negative sign above it, but due to computers and web browsers, I can't place this special notation properly in cyberspace, so don't get confused if you look this up in a mineralogy book! It should be pronounced as “negative 3” or “bar3”. In this instance, the -3 is the notation for the 3-fold axis of rotary inversion. I will consistently use the negative sign before the number when it is necessary. The same stands true for my notation when dealing with Miller indices or general form notation.

Crystallographers group forms by their symmetry notation, the first seven we will consider have the same symmetry - 4/m-32/m.

**CUBE**—The cube is composed of 6 square faces at 90 degree angles to each other. Each face intersects one of the crystallographic axes and is parallel to the other two (fig. 3.2). This form, {001}, is one of the easiest to recognize and many minerals display it with little modification. Think of galena, pyrite, fluorite, perovskite, or halite cubes!

**OCTAHEDRON**—The octahedron is a form composed of 8 equilateral triangles. These triangle-shaped faces intersect all 3 crystallographic axes at the same distance, thus the form notation of {111} (fig. 3.3). Minerals commonly exhibiting the simple octahedral form are magnetite, chromite, franklinite, spinel, pyrochlore, cuprite, gold, and diamond. Sometimes fluorite, pyrite, and galena take this form.

**DODECAHEDRON** (AKA Rhombic Dodecahedron) — This form is composed of 12 rhomb-shaped faces (fig. 3.4). Each of these rhomb-shaped faces intersects two of the axes at equidistance and is parallel to the 3rd axis, thus the notation {011}. The different mineral species of the garnet group often display this form. Magnetite and sodalite sometimes exhibit this form.
TETRAHEXAHEDRON—This form has 24 isosceles triangular faces. The easiest way to understand its shape is to envision a cube that on each face has 4 equal-sized triangular faces (fig. 3.5) that have been raised from the center of the cube face. Each triangular face has its base attached to the edge of the cube and the apex of the two equal-length sides rises to meet the 4-fold axis. Because of the variation of inclination to this axis, there exists a number of possible tetrahexahedral forms, but all meet the general notation of {0hl}.

The most common form is {012}. It is interesting to note that as the combination of each set of 4 faces rise along the axis, this form approaches the dodecahedron. As they fall, the form approaches a cube. The tetrahexahedron is rarely the dominant form on natural crystals, instead being subordinant to the cube, octahedron or dodecahedron (fig. 3.6). Cubic minerals on which you may sometimes see this form exhibited include fluorite (cube and tetrahexahedron), magnetite or copper (octahedron and tetrahexahedron) and garnet (dodecahedron and tetrahexahedron).

TRAPEZOHEDRON (AKA Tetragon-trioctahedron)—This form has 24 similar trapezium-shaped faces. If my Webster’s is correct, a trapezium is a 4-sided plane that has no sides parallel. Each of these faces intersects a crystallographic axis at a unit distance and the other two axes at equal distances, but those intersections must be greater than the unit distance. It sounds pretty complicated, but see the drawing (fig. 3.7). Because there may be various intercepts distances on the two axes, the form symbol {hhl} in general is used. The most common mineral form is {112}. Two silicate minerals, analcime and leucite, usually crystallize as simple trapezohedrons. This form is not uncommon, varying from dominant to subordinate, on many varieties of garnet, where it is often combined with the dodecahedron (fig. 3.8).

TRISOCTAHEDRON (AKA Trigonal Trisoctahedron)—This is another 24-faced form, but the faces are isosceles triangles. Each face intersects two crystallographic axes at unity, and the third axes at some multiple of unity; hence the form notation in general of {hll}.

To more easily visualize what a trisoctahedron looks like, first think of an octahedron. Each octahedral face is divided into 3 isosceles triangles by drawing 3 lines, each originating at the center of the octahedral face and reaching the 3 corners of that face. Repeat this operation for the remaining 7 faces on an octahedron and you have a trisoctahedron (fig. 3.9). As a dominant form, the trisoctahedron is scarce, most commonly being reported for diamond, usually as a subordinant form (fig. 3.10).

It has recently been demonstrated that trisoctahedral diamond is probably not a true crystal form (true crystal forms are growth forms), but instead a solution form caused by the differential dissolving of octahedral diamond during its transport from the mantle to the crust, but that’s another story altogether! As a subordinant form, it has been reported in combination with the octahedron for fluorite and magnetite and in combination with cube and octahedron on complex crystals of galena.

HEXOCTAHEDRON—This form has 48 triangular faces, 6 faces appearing to be raised from each face of a simple octahedron. These may be envisioned by drawing a line from the center of each of the 3 edges of an octahedral face, through the face center to the opposite corner. Repeat this for the remaining 7 faces of an octahedron and you have a hexoctahedron (fig. 3.11).
Just like the trisoctahedral form, this form is most often seen on diamond, where it is thought to represent a solution form derived from an octahedron, not true crystallization. With both the tris- and hexoctahedron, the faces are often curved, resulting in a near spherical shape. The combination of dominant dodecahedron and subordinant hexoctahedron is not uncommon for garnet (fig. 3.12)

We have 8 remaining forms in the isometric system to consider. The next 4 have the Hermann-Mauguin notation of -43m. These are the tetrahedron, tristetrahedron, deltoid dodecahedron, and hextetrahedron.

**TETRAHEDRON**-- The tetrahedron includes both a positive and negative form with the notation \{111\} and \{1-11\}, respectively. These are simple mirror images of one another. A tetrahedron is a 4-faced form, each face being an equilateral triangle.

Each face intersects all 3 crystallographic axes at the same distance. You may derive this form from an octahedron by extending alternate faces until they meet (this also shrinks the opposing set of alternate faces until they disappear).

Figure 3.13 displays the orientation of the tetrahedral form in relation to the cube. We aren't just speculating that both the positive and negative forms exist because they are often seen together (fig. 3.14) on a single crystal!

If both positive and negative forms are equal sized on a single crystal, then the initial appearance of the crystal form is INDISTINGUISHABLE from an octahedron. Here is where the differences in and orientation of surface features become exceedingly important in form study. One mineral so commonly has this crystal form that the mineral was named after the form itself - tetrahedrite. Other examples are diamond, helvite, and sphalerite.

**TRISTETRAHEDRON**-- By now, I think you might be able to tell me from my previous examples how to derive this form. Yep, take a tetrahedron and raise 3 isoceles triangle-shaped faces on each of the 4 tetrahedral faces. So this form has 12 triangular faces (fig. 3.15).

Just like the tetrahedron, there are both positive and negative forms, designated as \{hhl\} and \{h-hl\}, respectively. This is only a relatively common form on tetrahedrite, usually subordinant to the tetrahedron (fig. 3.16), but has also been reported on sphalerite and boracite. The possibility of it being present on diamond can’t be overlooked, but as mentioned, it may be the result of solution processes, rather than crystallization.

**HEXTETRAHEDRON**-- Again, we take a tetrahedron and, in similar manner as the hexoctahedron, we raise 6 triangular faces having a common apex from the center of the equilateral triangular face of the tetrahedron. Repeating this on the entire tetrahedron results in 24 faces (fig. 3.17). There are both positive and negative forms, designated as \{hkl\} and \{h-kl\}, respectively. This form has been reported on tetrahedrite, but rarely on sphalerite. Also a possible solution form on diamond.
DELTOID DODECAHEDRON—This is a 12-faced form, derived by raising 3 4-sided faces on each face of a tetrahedron (fig. 3.18). The shape of the resultant faces are rhombic. There are both positive and negative forms, designated as {hll} and {h-ll}, respectively. This form is sometimes seen as a subordinate one on tetrahedrite or sphalerite, where it would appear as a set of 3 rhombic faces modifying the corners of the dominant tetrahedral shape.

Now we have only 4 remaining forms to discuss in the isometric system. The first to consider is the gyroid.

GYROID (Pentagon-trioctahedron)—This form has no center of symmetry! The Hermann-Mauguin notation is 432. There are two forms, based on right- and left-handed symmetry (fig. 3.19). Older mineral textbooks state that this is a rare form, sometimes reported on cuprite. But most recent textbooks indicate that a restudy of cuprite’s crystallography showed cuprite to probably be hexoctahedral. If this is so, then we have no natural mineral that crystallizes with this form, although some laboratory-grown crystals with this form are known.

Two of the 3 remaining forms have 3 2-fold rotational axes, 4 3-fold rotary inversion axes, and 3 of the axial planes are mirror planes of symmetry. The Hermann-Mauguin notation is 2/m-3. These forms consist of the pyritohedron and the diploid.

PYRITOHEDRON (Pentagonal dodecahedron)—There are 12 pentagonal faces, each of which intersects one crystallographic axis at unity, intersects a second axis at some multiple of unity, and is parallel to the third axis. There are positive and negative forms, designated as {h0l} and {0kl}, respectively. There are a number of pyritohedral forms, differing due to the degree of inclination of the faces. The most common form is the {102}, the positive form (fig. 3.20). Pyrite is the only common mineral that displays this form. It is often subordinate, combining with the cube, diploid (below), or octahedron.

DIPLOID (Didodecahedron)—There are 24 faces (fig. 3.21), each face corresponding to one-half of the faces of a hexoctahedron. This is a rare form. You should compare figures 3.20 and 3.21. The diploid looks like a pyritohedron where two faces are made from each pentagonal face of the pyritohedron. The resulting faces are trapezia. There are both positive and negative forms, designated as {hkl} and {khl}, respectively. Pyrite is the only common mineral that exhibits the diploid form.

Believe it or not, we just reached the last form of the isometric minerals! If you are still with me at this point (mentally and physically), then I submit that you either 1) had little to do today to have the time to read this entire article, 2) are doing this to avoid starting a major project (like a term paper), 3) have a mental health problem and need serious counseling, or 4) are serious about learning more concerning Crystallography! So, let's grab this last form, the tetartoid (say it 10 times - real fast), and finish it off.

TETARTOID (pentagon-tritetrahedron)—The Hermann-Mauguin notation is 23. It has 3 2-fold rotation crystallographic axes and 4 diagonal axes that have 3-fold rotational symmetry. There are actually 4 separate forms in this class: positive right {hkl}, positive left {khl}, negative right {k-hl}, and negative left {h-kl}. See figure 3.22 for the positive right form. Cobaltite, an uncommon mineral, often crystallizes in this form. The tetartoid may be present as a subordinate form in combination with the cube, dodecahedron, pyritohedron, tetrahedron, and deltoid dodecahedron.

Do you remember the naturalist Steno from the 16th century?? We mentioned him in the introductory article. Well, here’s some information that he and later crystallographer’s discovered about isometric crystals and the interfacial angles of some of the different forms. This information could come in handy when you are deciding which of the common forms you have present on complex isometric crystals.

- The angle between two adjoining cube faces is 90 degrees.
- The angle between two adjoining octahedral faces is 70 degrees 32 minutes.
- The angle between two adjoining dodecahedral faces is 60 degrees.
- The angle between a cube (100) and a octahedron (111) is 54 degrees 44 minutes.
- The angle between a cube (100) and a dodecahedron (110) is 45 degrees.
- The angle between an octahedron (111) and a dodecahedron (110) is 35 degrees 16 minutes.
ALL RIGHT!! Whether you realize it or not, you have just gotten through the **most lengthy** discussion of crystal forms in this series. Therefore, I humbly bestow upon you the Order of the Golden Axial Cross! There will be other awards and accolades for stamina, perseverance, tenacity, and dedication as you proceed in your study of geometrical crystallography. Now if you are feeling a bit too spherical after digesting all of the isometric system, let’s shed some of our symmetry by exercising our mental powers on some lower symmetry forms in the next article!!

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**Part 4: The Tetragonal System**

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